Case Study – Hyperelastic Material Parameters

PROBLEM

Hyperelastic materials (rubbers, elastomers, certain plastics) are commonly used engineering materials, applied widely in automotive, aerospace, engineering, and biomedical industry. Though many material models such as Mooney-Rivlin, Arruda-Boyce and Ogden are famous for such "hyper-elastic" materials, the theory and practice of how to evaluate material model parameters is underdeveloped. Having such theory and practice would improve testing, formulation and quality systems.

In this project, we suggested a theory for evaluating material parameters for the Mooney-Rivlin model.

THEORY

Mooney-Rivlin model. The Mooney-Rivlin hyper-elastic model is given by the energy density function

$$W = C_1(I_1-3)+C_2(I_2-3)$$

Uniaxial stretch. From the above, the force for uniaxially stretching a cylinder is calculated to be

$$F = 2\pi r_1^2\left(C_1\left(\frac{l_2}{l_1}-\frac{l_1^2}{l_2^2}\right)+C_2\left(1-\frac{l_1^3}{l_2^3}\right)\right)$$

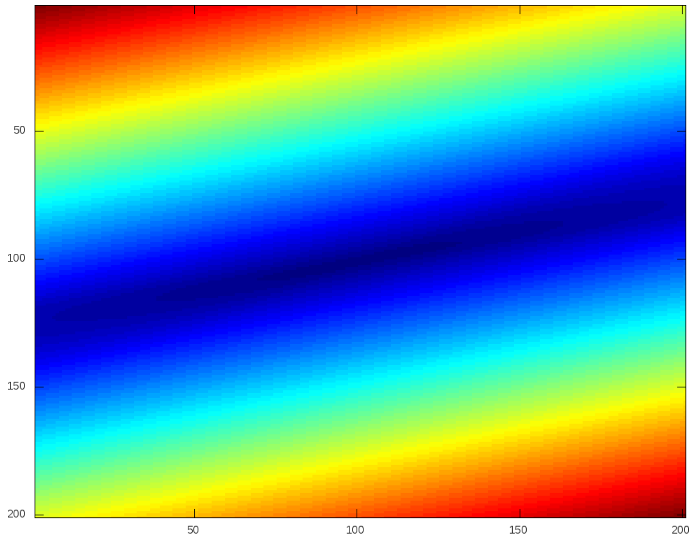
Biaxial stretch. Force for stretching a cylindrical disc biaxially is calculated to be

$$F = 4\pi l_1 r_1\left(C_1\left(\frac{r_2}{r_1}-\frac{r_1^5}{r_2^5}\right)+C_2\left(\frac{r_2^3}{r_1^3}-\frac{r_1^2}{r_2^2}\right)\right)$$

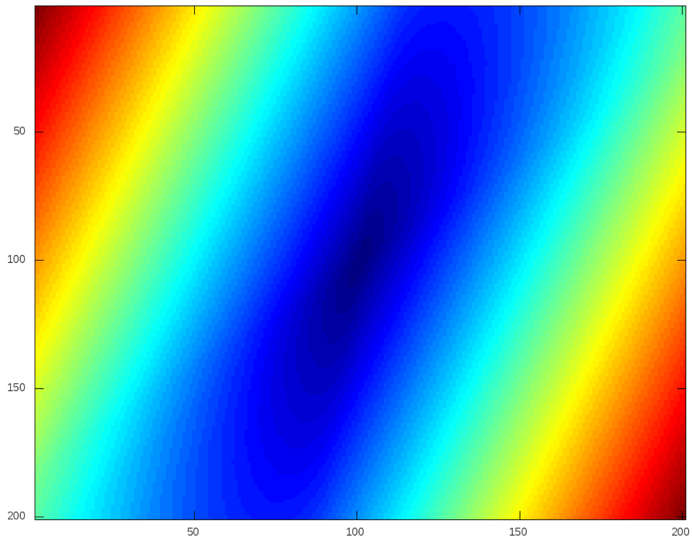
INSIGHTS

Confusion curves. A simplistic attempt is to perform a uniaxial test of the material, and find C_1 and C_2 that best fit the data. The confusion level sets of this methodology show that a long but thin confusion manifold exists in this methodology (see diagram).

Thankfully, the confusion diagram for a biaxial test is oriented quite differently than that of a uniaxial test. Performing both the tests and fitting the data will give accurate results for both C_1 and C_2.



Confusion curves for uniaxial test



Confusion curves for biaxial test

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